Logarithm Rules

Conversion

To convert between the exponential and the logarithmic forms transfer corresponding elements from one to the other.

Exponents: $\text{Base}^{\text{Exponent}} = \text{Result}$ Logarithm: $\text{Log}_{\text{Base}}(\text{Result}) = \text{Exponent}$

Example:

$5^2 = 25$ | Base = 5, Exponent = 2, Result = 25 | $\text{Log}_5(25) = 2$

Trick: Follow the arrow and rewrite elements in the new order.
Rewrite b then y then x.

$\text{Log}_b(x) = y \rightarrow b^y = x \qquad b^x = y \rightarrow \text{Log}_b(y) = x$

Properties

For the general case where the letters m, n and b are positive numbers.

1. $\text{Log}_b(b) = 1$ --- Log to base 'b' of 'b' = 1.

   Example: $\text{Log}_5(5) = 1$

2. $\text{Log}_b(1) = 0$ --- Log to any base of 1 = 0.

   Example: $\text{Log}_5(1) = 0$

3. $\text{Log}_b(m \cdot n) = \text{Log}_b(m) + \text{Log}_b(n)$ --- Product Property

   Example: $\text{Log}_{10}(4 \cdot x) = \text{Log}_{10}(4) + \text{Log}_{10}(x)$

4. $\text{Log}_b\left(\frac{m}{n}\right) = \text{Log}_b(m) - \text{Log}_b(n)$ --- Quotient Property

   Example: $\text{Log}_{10}\left(\frac{6}{z}\right) = \text{Log}_{10}(6) - \text{Log}_{10}(z)$

5. $\text{Log}_b(r^p) = p \cdot \text{Log}_b(r)$ --- Power Property

   Example: $\text{Log}_{10}(x^2) = 2 \cdot \text{Log}_{10}(x)$

6. $\text{Log}_a x = \frac{\text{Log}(x)}{\text{Log}(a)} = \frac{\text{Ln}(x)}{\text{Ln}(a)}$ --- Change of base

   Example: $\text{Log}_5(x) = \frac{\text{Log}(x)}{\text{Log}(5)}$

Caution!

The 'result' in the logarithm can never be negative; $\text{Log}_5(-25)$ is NOT valid.