Graphing Rational Functions

- ✓ Rational functions: Equations of the form $f(x) = \frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are both polynomials.

- ✓ y intercept: This is the point where the curve crosses the y axis. Set x = 0 and solve for y.

- ✓ x intercept: This is the point where the curve crosses the x axis. Set the numerator equal to zero and solve for x.

- ✓ Vertical Asymptotes: Find the values of x which make the denominator equal to zero.

  *Warning: The function must be in lowest terms! **Why?***

- ✓ Horizontal Asymptotes:

  1. If the degree of numerator is less than degree of denominator, the graph has a horizontal asymptote y = 0. **Why?**

  2. If the degree of numerator equals the degree of denominator then the horizontal asymptote is equal to the ratio of the leading coefficients.

     Example: $y = \frac{5x^2 + 6x - 7}{3x^2}$ Horizontal asymptote $y = \frac{5}{3}$

- ✓ Slant Asymptotes:

  1. If the degree of the numerator is one more than the degree of the denominator, perform the long division $P(x) \div Q(x)$. The quotient is the slant asymptote. *(We ignore the remainder).*

Warning: The rational function must be fully reduced **before** asymptote analysis takes place!

Example:

Graph $g(x) = \dfrac{(x - 4)^2}{(x - 2) \cdot (x - 6)}$

- x - intercepts – (4, 0)
- y - intercepts – (0, 4/3)

- vertical asymptotes – The lines $x = 2$ and $x = 6$ are the vertical asymptotes.
- horizontal asymptotes – Degree of the numerator equals the degree of the denominator → The horizontal asymptote is the ratio of the leading coefficients.

  The horizontal asymptote is y = 1 (from 1/1).

Graph other points as needed….